Opting-out in profit-sharing regulation*

Michele Moretto† and Paola Valbonesi‡

Abstract

To avoid the extremely high profit levels found in recent experiences with price cap regulation, some regulators have proposed a profit-sharing mechanism that revises prices to the benefit of consumers. This paper investigates the conditions under which a regulator can implement such a profit-sharing scheme, having the option to revoke the contract if the firm's profits are excessive.

When this option is included in the regulator's objective function and the cost of exercising it is not too high, a long-term equilibrium arises with a state-contingent sharing rule that guarantees an appropriate level of profits. The model determines both the level of profits that triggers the profit-sharing mechanism and the consequent price adjustment endogenously. There is an endogenous regulatory lag initially characterized by a price cap regulation, followed by a period of profit-sharing regime where the firm is motivated to cut prices to avoid revocation.

Key words: Public utilities, Price cap regulation, Profit-sharing, Stochastic games.

JEL: C73, L33, L51

---

*This paper circulated as mimeo with the title: "Is there hope for profit-sharing regulation?". We wish to thank Alberto Bennardo, Cesare Dosi, Gianni De Fraja, Andrea Gamba, Antonio Nicolò, Carlo Scarpa, Pierre Régibeau and seminar participants at Naples, Brescia, Padua and Verona Universities for their useful comments. Both the authors acknowledge financial support from MIUR through grant Cofin 2002 - No.2002131535_004.

†Dipartimento di Economia, Università di Brescia, Via S.Faustino 74b, 25122 - Brescia, Italy. E-mail: **moretto@eco.unibs.it**

‡Dipartimento di Economia, Università di Padova, Via del Santo 33, 35100 - Padova, Italy. E-mail: **paola.valbonesi@unipd.it**

1 Introduction

Regulators dislike high corporate profits under price-cap regulation (PCR) because they can reduce consumers' welfare and - favouring the firm - downgrade the regulator's own reputation for being able to set the "correct" price cap. Recent British and US experience in regulation of public utilities shows that price cap allows prices to diverge greatly from actual costs and generate "abnormal" profits for the firm. This drawback of the price cap as an incentive mechanism stems from its inability to set a contingent price that incorporates all the uncertainties faced by the firm in each period of the regulatory contract.

In real life, pure PCR has been modified in a variety of ways to induce the regulated firms to rebate part of the profits to their customers. "Profit-sharing" (PS) schemes - which is how these modifications are usually described - are introduced in the PCR to make the real price depends on the realised level of profit. Sappington (2002), for example, shows that the modifications of pure PCR in the US telecommunication industry are usually set as: a) direct payments to customers or b) reductions in prices of key services.

In this paper we mainly deal with case b). We take the endogenous rise of a PS rule in a long-term franchise contract between a public utility and a regulator, where the former is the residual claimant of the difference between the regulated price and costs and the latter remains residual decision maker.[1]

Unlike most of the recent literature on PS, which concentrates on efficiency[2] and views PS schemes essentially as actions by the regulator without technically breaching the PCR commitment[3], this paper investigates the conditions under which the regulator can induce the firm to accept and follow a PS scheme introduced by unilateral decision of the regulator to favour customers.[4] In particular, if after having contracted a PCR the regulator finds

[1] The regulator's role as residual decision maker can also be related to the need to protect the customers' "right to be served". See Goldberg (1976).

[2] The fundamental trade-off generated by the introduction of a PS clause is between lowering extreme profits and dulling the firm's incentive for cost reduction. The literature has stressed that compulsory sharing of profit may reduce the firm's incentive to minimize operating costs and increase revenue (Lyon, 1996; Crew and Kleindorfer, 1996); it may provide an incentive to undertake projects that are unduly risky (Blackmon, 1994) and, finally, it may lead the utility to delay investment (Moretto et al., 2003).

[3] See Weisman (2002).

[4] This happened for instance in 1995 when the British electricity regulator - realising that its previous intervention on prices had been too mild - intervened on prices well before

that the firm is making "excessively" high profits, he introduces a sharing rule to minimize the social welfare loss and threatens to revoke the contract if the firm does not comply. Contract closure is then an "outside" option the regulator can wield to compel the firm to comply with a PS mechanism.[5] As the regulator has the power but not the obligation to intervene, we model his outside option as a perpetual call option, with the firm's value as underlying asset. It follows that the regulator must determine the time at which to pay an exercise cost for the managment of a project whose value is stochastic. This exercise cost captures both direct and indirect costs of contract closure: while indirect costs refer mainly to welfare loss, direct costs, reflect training and hiring new personnel and/or adopting new technologies to provide the service, searching for a new franchisee, or legal expenditures if the firm decides to sue the regulator.[6]

Our results show that when contract closure is the regulator's only outside option[7] and he can credibily threaten revocation, then the equilibrium is self-enforcing, sustainability of regulation is reached and a state-contingent proportional PS rule endogenously determined. However, as revocation is costly, there may be a stochastic regulatory lag during which prices are not revised and it is not optimal for the regulator to exercise his option. The higher the costs of revocation, the longer the regulatory lag and the less the sharing.

On a formal level, our paper builds upon two distinct strands in the literature. One relates to the stochastic control techniques recently developed to identify optimal timing rules and optimal barrier regulations,[8] which are

---

the price review due in 1999. Another well-known real example is Oftel - the British tlc regulator - which in 1991 unilaterally raised the X factor in the PCR for British Telecom far ahead of the scheduled review. On this see Amstrong et al.(1994, p.227-228).

[5]Contract revocation can be seen as an extreme punishment: the management of the utility is put back in the hands of government, which can then choose direct management, privatization, or contracting out to another firm.

[6]Costs of revocation are higher, the largest is the firm ability to determine regulatory capture. For a discussion of regulatory capture see Laffont and Tirole (1994, ch.11).

[7]We do not consider other regulator's outside options as - for example - taxation of excess profit. A well-known example of such a power is the "windfall tax" adopted in the UK in 1997 and levied on the privatised utility companies. The idea was to claw back some of the excess profits due to the underpricing of original privatizations and under-regulation in the first few years. For a discussion on the distortions in terms of efficiency, equity and administrative feasibility, see Chennells (1997).

[8]See Harrison and Taksar (1983), Harrison (1985).

widely used in the literature of irreversible investments (Dixit and Pindyck, 1994), and emphasize the option value of delaying investment decision, i.e. the value of waiting for better (although never complete) information. The second considers the existence of efficient sub-game perfect equilibria for infinite-horizon threat-games where, in the absence of a binding commitment, if the threatened act benefits the threatener it is an equilibrium for the victim to make a stream of payments over time (Klein and O'Flaherty, 1993; Shavell and Spier, 1996). The expectation of future payment keeps the threatener from making good on his threat.[9]

The plan of the paper is as follows: Section 2 defines PCR and PS in a simple reduced-form framework. Section 3 describes the regulatory game, introducing first the timing and the regulator's objective function and then the optimal revocation and the regulatory equilibria. Section 4 discusses results and policy implications, in a comparison with the previous literature. A very few final remarks conclude, followed by an Appendix containing all the proofs.

2 The firm's profit function and PS

In this section we model the firm's profit function in providing a public utility under a PCR. Our framework is very general: the profit function is modelled in reduced form as its expected change over time is affected only by market conditions and the parameters of the price cap regime.

As for the standard price cap rule, we assume that the price is allowed to increase by the difference between expected inflation (the Retail Price Index, RPI) and an exogenously given expected increase in productivity over time (x). Formally:

$$dp_t = (RPI - x)p_t dt, \qquad \text{with } p_0 = p \tag{1}$$

When the provision of the utility begins, the single project gives a cash flow at each time t expressed by:

$$\pi(p_t, q_t) = p_t \theta_t \tag{2}$$

[9] See Moretto and Rossini (2003) for an application of a continuous time threat game to design a severance payments to prevent plant closure by shareholders.

where, for the sake of simplicity, the operating costs are set to zero[10] and θ_t denotes the quantity demanded. Uncertainty is captured by assuming that θ_t evolves over time by a trendless, geometric Brownian motion, with instantaneous volatility $\sigma \neq 0$. That is:

$$d\theta_t = \sigma\theta_t dW_t, \quad \theta_0 = \theta \tag{3}$$

where dW_t is the standard increment of a Wiener process, uncorrelated over time and satisfying the conditions that $E(dW_t) = 0$ and $E[(dW_t)^2] = dt$.

These assumptions enable us to reduce the model to one dimension. Expanding $d\pi(p_t, \theta_t)$ and applying Itô's lemma it is easy to show that $\pi(p_t, \theta_t)$ is driven by the following geometric Brownian motion:

$$d\pi_t = \alpha\pi_t dt + \sigma\pi_t dW_t \quad \text{with } \pi_0 = \pi, \tag{4}$$

where by the assumption of a completely inelastic demand function, the drift parameter of π_t is simply $\alpha \equiv (RPI - x)$.[11]

When the regulatory scheme (1) allows the firm to retain huge profits, the regulator forces the firm to share its "excess" profits with consumers by imposing a price-reduction mechanism. As Sappington (2002) shows, there are a number of ways of introducing profit-sharing; the one we consider here is to set an upper bound on profit, π^*, above which a higher x-factor applies. A PS rule is thus defined as a modification of (1) as follows:[12]

$$dp_t = (RPI - x_j)p_t dt \;\; \text{where } x_j = \begin{cases} x & \text{if } \pi_t < \pi^* \\ x' & \text{if } \pi_t \geq \pi^* \end{cases} \tag{5}$$

with $x < x'$.

The modification of PCR by PS as in (5) requires one to define: a) the level of profits π^* and b) the difference $x' - x$. This is the subject of the next section.

[10] This avoids the need to consider such operating options for the firms as reducing output or even shutting down, and thereby considering reducing variable costs. The presence of operating options raises the value of the firm. See Macdonald and Siegel (1986) and, for a thorough discussion, Dixit and Pindyck (1994), chs. 6 and 7.

[11] In this case the monopolist's profits are increasing in price. This form of the demand function concords with Joskow and Schmalensee (1986, p.3) who note that the demand for electricity, water and gas from most industrial customers and all residential customers is highly inelastic, especially in the short-run.

[12] See Sappington and Weisman (1996) and Schmalensee (1979) for qualitatively analogous rules.

3 The regulatory game

In this section we investigate the continuous time threat-game between a regulator and the regulated utility. After a simple description of the sequence of the moves, we define the regulator's objective function - in terms of social losses - and the value of the option to revoke. We analyse the regulatory equilibria, first in a simple discrete time example and then for the general case. For the latter we derive a sub-game perfect equilibrium that belongs to the class of efficient perfect equilibria in pure strategies.

3.1 The sequence of moves

At $t = -\hat{t}$ a risk-neutral profit-maximizing firm is delegated to manage a one-time sunk indivisible public project under a PCR. For this simple contract we assume the regulator retains ownership of the asset while the firm has the "right to use it" (i.e. franchise contract).[13] We further assume that no new investment is made during the contract period.[14]

At $t \geq 0$,[15] the firm has began to provide the utility; if the regulator finds that the firm is making "excessively" high profits, he announces a profit ceiling, say π^*, above which the gains must be shared. And the regulator flanks the announcement by threatening to revoke the contract if the firm does not comply.

When at $t = T^*$ the firm's profits cross the threshold π^*, the firm may either cut prices to keep its profits below π^* and thus continue to provide the utility or else keep its profits unchanged, knowing that as a consequence the contract could be revoked.

The new lower price remains valid until profits again go above the trigger

[13] In principle, our analysis could be applied to utilities of global range (national utilities) but given our assumption on the regulator's revocation of the contract, the local dimension of the public utility (i.e.sewage managment and water supply, urban waste disposal and public transport) seems more realistic. In fact, the management of a contract at national level can affect the firm's bargaining power, which in turn can affect the revocation decision (regulatory capture). For a discussion of the regulation of local public utilities and the regulator's revocation of the contract, see also Moretto and Valbonesi (2000)

[14] This avoids complications due to indemnities in the case of revocation. See Section 4 for further comments.

[15] It will be evident below that by the Markov Property of (4), in our model it is not important when the regulator annuonces π^* as long it is after the PCR is set. For simplicity we call this time zero.

level, promoting another revision. Thus, the game involves a regulatory lag, which lasts until the firm adjusts its profits according to the sharing rule.

3.2 The regulator's objective function

Since the firm is risk-neutral, using the simplified expression for the profit function (4), we can write the market value of the project as:[16]

$$V(\pi_t) = \frac{\pi_t}{\rho - \alpha} \equiv \frac{p_t \theta_t}{\rho - \alpha} \tag{6}$$

where $\rho > \alpha$ is the constant risk-free rate of interest.[17] Thus V being a costant multiple of π, it also follows a geometric Brownian motion with the same parameters $\alpha \equiv (RPI - x)$ and σ, i.e.:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t, \quad V_0 = V \tag{7}$$

Therefore, choosing π^* is equivalent to choosing an upper limit to the value of the firm V^*. This means that the analysis could be readly be replicated using the present value as the state variable. Hereinafter we may take V_t as the primitive exogenous state variable for the regulatory process.

Any increase in the value of the firm may reduce the monetary value of social welfare; accordingly, when this reduction is perceived as excessively high, the regulator revokes the contract. However, revocation is costly. Hence, we may set the regulator's objective function to be minimized:

$$-G(V_t) \equiv L(V_t) + (I - V_t) \tag{8}$$

where $L(V_t)$ is the welfare loss due to revoking the contract at time t, with $L'(V_t) > 0$ and $L(V_0) \geq 0$, and $I - V_t$ is the net cost of exercising the option.

[16] The market value of an infinite lived project can be evaluated as the expected present value of discounted cash flows (see Harrison, 1985, p. 44).

$$V(\pi_t) = E_t \left\{ \int_t^\infty e^{-\rho s} \pi_s ds \right\} = \frac{\pi_t}{\rho - \alpha}$$

where E denotes the real-world expectation operator and ρ is the risk-free at which future cash flows are discounted.

[17] Alternatively, we could use a discount rate that includes an appropriate adjustment for risk and take the expectation with respect to a distribution for π that is adjusted for risk neutrality (see Cox and Ross, 1976; Harrison and Kreps, 1979).

Among the possible many ways of modeling the welfare loss, we adopt a utilitarian criterion and approximate it as $L(V_t) = \lambda(V_t - V_0)$, where λ is the opportunity cost of direct management by the regulator ($\lambda > 0$ due to fiscal distortion in raising public funds to run the service) and V_0 is the value of the utility at time zero.[18]

3.3 Optimal revocation

As noted, the regulator revokes the contract if the social loss is perceived as too large. However, minimizing (8) is equivalent to maximize $G(V_t) \equiv V_t - I - L(V_t)$ which makes it evident that rent extraction can be part of the regulator's purpose in revoking the contract (Crew and Kleindorfer, 1996, p.218). Excercising this option requires the payment of a sunk cost I plus a social cost $L(V_t)$. By the sunkness of I it is never optimal to revoke when $V_t - I - L(V_t)$ is zero, it is better to wait until the value reaches a higher level.

Defining $F(V)$ as the value of the option at $t = 0$, we get:

$$\begin{aligned} F(V) &= \max_T E_0 \left[(V_T - I - L(V_T)) e^{-\rho T} \mid V_0 = V \right] \\ &= \max_T E_0 \left[(1-\lambda) V_T - (I - \hat{V})) e^{-\rho T} \mid V_0 = V \right] \end{aligned} \tag{9}$$

where $\hat{V} \equiv \lambda V_0$, $T(V^*) = \inf(t \geq 0 \mid V_t - V^* = 0^+)$ is the unknown future time when the option is exercised and V^* is the threshold value that triggers that action.[19] The optimization is subject to (7) and V_0. To simplify discussion, we assume that $V_0 < V^*$ so that $T^* > 0$ (see the Appendix A.2 for the general case). Moreover, for the optimal revocation to make sense, we must also assume that $I - \hat{V} > 0$ and $V^* - I > 0$.

[18] According to the utilitarian criterion and assuming the totally inelastic demand, we can approximate the welfare function at time t by the weighted and discounted average of the net surplus of consumers $K - (1+\lambda)V_t$ and the value of the project V_t. Hence, the social loss is simply given by:

$$L(V_t) \equiv K - \lambda V_0 - [K - \lambda V_t] = \lambda(V_t - V_0)$$

where K is the expected value of the consumers' willingness to pay for the service (Laffont and Tirole, 1994).

[19] In the range $[V_0, V^*]$ the probability that V_t reaches V^* is (Cox and Miller, 1965, pp.

Note that $F(V)$ represents a perpetual call option. By using standard results in (real) option valuation[20] it is easy to rewrite (9) as:

Proposition 1 *The value of the option to revoke at any time $t \geq 0$ is:*

$$F(V_t) = \begin{cases} AV_t^{\beta_1} & \text{for all } V_t < V^* \\ \\ (1-\lambda)V_T - (I - \hat{V}) & \text{for all } V_t \geq V^* \end{cases} \tag{10}$$

where:

$$V^* = \frac{\beta_1}{\beta_1 - 1}\frac{1}{1-\lambda}(I - \hat{V}), \quad \text{with} \quad \frac{\beta_1}{\beta_1 - 1} > 1 \tag{11}$$

and:

$$A(V^*) = \frac{1-\lambda}{\beta_1}(V^*)^{1-\beta_1} > 0, \tag{12}$$

Proof. see Appendix A.1 ■

The regulator's optimal break-even rule is of the form: "**Revoke the contract as soon as the value of the project exceeds the adjusted break-even value** V^*".

There are two cases that are instructive here. First, if $\lambda \to 0$ then according to the utilitarian criterion the regulator is socially "indifferent" between direct management and franchising. From (11), V^* drops, increasing the probability of revocation. A second interesting case is when the opportunity cost of direct management by the regulator rises, i.e. $\lambda \geq 1$. In this case $V^* \to \infty$ and the regulator never revokes.[21]

232-234):

$$\Pr(T^* \leq +\infty) = \begin{cases} 1 & \text{if } \mu > 0 \\ \left(\dfrac{V_0}{V^*}\right)^{-2\mu/\sigma^2} & \text{if } \mu \leq 0 \end{cases}$$

where $\mu \equiv (\alpha - \sigma^2/2)$. Starting at V_0 in the interior of the range $(0, V^*]$, the process hits the trigger V^* if the trend $\alpha \equiv (RPI - x)$ is positive and sufficiently large with respect to the uncertainty. The process may drift away and never hit V^*, if α is positive but low with respect to the uncertainty (i.e. $\mu < 0$). In this last case revocation never happens.

[20] See Harrison (1985) and Dixit and Pindyck (1994).

[21] A welfare function in which the consumers' surplus is obtained from a demand function that is not completely inelastic would identify a $V^* < \infty$ also with $\lambda = 0$. This complicates the model but does not produce any useful additional results.

We can pinpoint the timing of revocation by comparing the opportunity cost of exercising the option with corresponding benefits of optimally postponing the decision. This can be done by evaluating the difference $F(V_t) - G(V_t)$ where, by (8), $G(V_t) \equiv (1-\lambda)V_t - (I - \hat{V})$ is the regulator's net benefit when the utility is expropriated at time t, and $F(V_t) = AV_t^{\beta_1} > 0$. If we assume $V_t < V^*$ so that the regulator finds it optimal to wait, we get:

$$F(V_t) - G(V_t) = (I - \hat{V}) + AV_t^{\beta_1} - (1 - \lambda)V_t \tag{13}$$

The first term on the r.h.s. of (13) is the direct cost of revocation the contract. The second term is the value of the option. Since "killing" the option is irreversible it appears as an opportunity cost. Finally, the third term is the value of the firm; as revocation reduces the welfare loss, in (13) it appears as the opportunity benefit. Given $V_t < V^*$ and $F(V_t) - G(V_t) > 0$, the opportunity benefit is less than the opportunity cost, so the decision to revoke the contract should be postponed.

3.4 The regulatory equilibria

As we have shown, the regulator has no incentive to revoke as long as V_t is less than V^*. Indeed, as $AV_t^{\beta_1} - [(1 - \lambda)V_t - (I - \hat{V})] > 0$ for all $V_t < V^*$, revocation carries a cost to the regulator that makes it not optimal; that is, the threat of revocation is not credible. On the other hand, for $V_t > V^*$ it is optimal; the threat of revocation is credible. This reveals the simple stationary nature that characterizes this extreme threat: as soon as V hits V^*, the contract is revoked the firm suffers the loss V^* and the regulator receives $V^* - I$.

To avoid revocation, the firm may be willing to reduce profits to keep V_t below V^*. However, without the binding commitment of the regulator any number of firm's profit reduction, based on the difference $V_t - V^*$, will be inefficient. The regulator's problem is that for $t \geq T^*$ he will always have an incentive to carry out his threat even if the firm reduces its profits. Since this means that the firm will not ward off the threat by reducing its profits, it will not reduce.[22] Furthermore, by backward induction, the same goes for any finite number of profit reductions. The regulator cannot overcome this problem in a single (or finite) period setting, and in this version of the model,

[22] See Klein and O'Flaherty (1993); Shavell and Spier (1996).

his threat will fail. The unique sub-game perfect equilibrium is inefficient: revocation is carried out regardless of the firm's positive net present value.

To avoid this inefficiency the firm must "control" its profits in continuum. For $t \geq T^*$ the firm sets V^* as its ceiling and reduces its expected profits by lowering the price cap just enough to keep V_t from crossing the ceiling V^*, so that the regulator is indifferent between continuing the contract and revoking it; that is, $F(V_t) = 0$ for $t \geq T^*$.

A discrete-time example

To get a sense of the properties of this sub-game perfect equilibrium, let us look at a simple discrete-time threat game between regulator and firm. To focus on the basic question at issue here and to make it simple, we impose the following restrictions:

1. The regulator sets V^* and makes the extreme threat of revoking the contract as soon, say at time T, as $V \geq V^*$;
2. The firm's value remains constant at $V \geq V^*$, for all $t \geq T$;
3. Time is discrete, i.e. $t \in \mathbb{N}_+$ and $T \in \mathbb{N}_+$;
4. If the revocation is carried out, the firm suffers a one-time loss V, and the regulator gains an asset of value $V - I$. Revocation is inefficient, i.e. $V > V - I$.
5. A discount factor $\delta \in [0, 1)$ is known to both the players.

This simple framework fits into the basic Shavell and Spier (1996) mode quite well. At any time $t \geq T$ the firm may choose to make a payment $r_t \geq 0$, upon which the regulator decides whether or not to revoke the contract. If he does revoke, the relationship is terminated, the firm suffers a loss V and the regulator obtains $V - I$.[23] If he does not revoke, the game goes ahead to period $t + 1$ and is repeated.

There are many efficient sub-game perfect equilibria for this discrete time game where the threat of revocation induces an infinite flow of payments by

[23] In addition, consumers gain the whole stream of past payments r_s, where $T^* \leq s \leq t$.

the firm to prevent contract closure.[24] In particular, if the regulator uses revocation as the most severe possible punishment, a set of efficient equilibria with a constant payment stream over time follows:

$$r_t = r = a(V)V \quad \text{where} \quad a(V) \in \left[\frac{1-\delta}{\delta}(\frac{V-I}{V}), \ (1-\delta)\right] \tag{14}$$

As is usual in these games, the regulator must believe that the payments, from the initial date and state (T, V), will continue forever. If the firm deviates by paying less than r, the regulator concludes that the firm will pay nothing in the future and revokes the contract immediately.

Although there are infinite divisions of the surplus $V-(V-I)$ in equilibrium, it is easy to show that the present value of the flow of payments $R(V)$ is:

$$R(V) \equiv \sum_{i=t}^{\infty} \delta^{i-t} r \leq V \tag{15}$$

with the equality for the upper bound $r = (1-\delta)V$, when the regulator is able to extract the entire surplus. In the specific, if we set $\delta = (\rho - \alpha)$ the per-period payment is just the firm's per-period profit π.[25]

The general case

Let us now go back to the general case. If the firm starts with the initial value V_0, the optimal stationary strategy becomes:[26]

- for $V_t < V^*$, V_t is allowed to follow the geometric Brownian motion (7);

[24] See Shavell and Spier (1996, Proposition 2).

[25] Since $e^{-(\rho-\alpha)\Delta t} = 1 - (\rho - \alpha)\Delta t$, we may write:

$$\sum_{i=0}^{\infty}(\rho-\alpha)^i r = V = \frac{\pi}{\rho-\alpha} = \int_0^{\infty} \pi e^{-(\rho-\alpha)t} dt$$

so that $r = (1-(\rho-\alpha))V = \pi$

[26] We deal here only with pure strategies and keep the exposition at a heuristic level referring to the Appendix A.2 for the proof of the regulatory scheme and its properties. Our choice of pure strategies can be justified within the context of this paper by the fact that they are simple, requiring the firm and the regulator to have a very low level of rationality.

- at V^*, a costless "profit control" dr_t is applied to stop V_t from going above V^*.[27]

The overall process can be described as:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t - dr_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \tag{16}$$

where the increment dr_t gives the amount that the firm is willing to pay (i.e. the profit reduction it is willing to accept) between t and $t + dt$ to keep the contract alive. We can summarize the resulting regulation in the following proposition:

Proposition 2 (the regulatory equilibria) *For any $V^* > V_0 > 0$, if the firm regulates its profits with the non-decreasing proportional rule:*

$$r_t = \; a(V^*)V_t \qquad \text{if } V_t \geq V^*, \qquad\qquad \text{where } \; a(V^*) \equiv [1 - \inf_{T^* \leq v \leq t} \left(\frac{V^*}{V_v}\right)], \tag{17}$$

then the following regulator strategy represents a sub-game perfect equilibrium:

$$\phi(V_t, r_t) = \begin{cases} \textbf{\textit{Do not revoke}} \\ \text{at } t = T^* \text{ if the firm has followed rule } r_t \\ \text{to keep } V_t < V^* \text{ for } t' < t \\ \\ \textbf{\textit{Revoke}} \\ \text{if the firm has deviated from } r_t \\ \text{at any } t' < t \end{cases}$$

Proof. see Appendix A.2. ■

As highlighted in Proposition 2, our stochastic-continuous time framework calls for a refinement of the threat strategy used by Shavell and Spier (1996). This strategy implies an instantaneous response by the regulator when the firm departs from profit rule (17), with the most severe punishment.[28]

[27] The assumption that the profit control is cost-free is not technically necessary to our analysis. Assuming that the firm faces a cost $C_t = cdr(V_t)$ does not alter the results.

[28] As we know, in continuous time repeated games there is no notion of *last time before* t, so induction cannot be applied. We refer the reader to Simon and Stinchcombe (1989) and Bergin and MacLeod (1993) for examples of how to represent continuous time as a sequence of discrete-time grids that becomes infinitely negligible.

The optimal profit regulation r_t is still proportional to the firm's value (see Figure 3 in the Appendix A.2).[29] According to strategy rule ϕ, the firm observes V_t and chooses an action (17), and the regulator stays with ($\phi(V_t, r_t) =$ "Do not Revoke" for all $t \geq T^*$) or, equivalently, at T^*, sets a continuous time control rule for each realization of V_t for any $t \geq T^*$.[30] The value of the firm under profit regulation is obtained from V_t by imposition of an upper control barrier at V^*; r_t increases to keep V_t lower than V^* and is given by the cumulative amount of profit control exerted on the sample path of V_t up to t. It follows that regulation relates to the history of the game and past value realizations, which makes $\phi(V_t, r_t)$ a time-dependent strategy. The regulator's threat strategy is adopted if the firm deviates from the regulation rule (17). The regulator believes that this mechanism, from the initial date and state (T^*, V^*), will be retained for the whole (stochastic) planning horizon. If the firm deviates, the regulator expects a fresh rule: a change in the profit regulation policy is perceived by the regulator as a stoppage of regulation. The punishment for deviation is revocation of the contract. Since the project is infinitely-lived, the present value of forgone profits will ensure participation by the firm, while the expectation of future profit regulations keeps the regulator from carrying out his threat.

As in (15), we can define the present value of the flow of payments. Denoting the expected value of the firm's future cumulative profit reduction due to the regulation by $R(V; V^*)$, we get (see Appendix A.2):

$$R(V_t; V^*) \equiv E_t \left\{ \int_t^{\infty} e^{-\rho(s-t)} dr(V_s) \mid V_t \in (0, V^*] \right\} = B(V^*) V_t^{\beta_1} \leq V_t \tag{18}$$

where $B(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0$. Direct inspection of (18) and (10) shows that in equilibrium the welfare losses are exactly offset by the benefits from profit regulation, i.e.:

$$A(V^*)V_t^{\beta_1} - (1-\lambda)B(V^*)V_t^{\beta_1} = 0$$

and the regulator's option to revoke the contract drops to zero (see Appendix A.2).

[29] In technical terms, V^* is no longer an absorbing barrier but a (reflecting) barrier control, while the optimal control r_t is a right-continuous, non-decreasing and non-negative adapted process (Harrison and Taksar, 1983; Harrison, 1985).

[30] In our continuous time setting we assume, without any loss of generality, that when the regulator is indifferent it does not exercise the option.

Finally, although the public project lives forever, profit is adjusted within a finite (stochastic) time span. Owing to uncertainty, neither firm nor regulator can perfectly predict V_t each time. As V_t follows a random walk, for each time interval dt there is a constant probability of moving up or down, i.e. of the game continuing one more period. Therefore the game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite. This can be summarized in the following proposition:

Proposition 3 (the timing of regulation) *As long as $V_t < V^*$ nothing is done. As soon as V_t crosses V^* from below the firm adjusts its profits, using (17) to keep the regulator indifferent to revocation and $F(V_t) = 0$. Adjustment goes on up to the point where the unadjusted firm's value V_t crosses the trigger V^* from above and the regulator becomes (again) indifferent.*

Proof. see Appendix A.2. ■

Although the firm prefers to adjust rather than terminate the contract (i.e. the loss from closure is greater than the expected cost of price adjustment), it always prefers to stop the adjustment if the threat of revocation is not carried out. However, since the regulator's strategy is time-dependent, the firm cannot decide whether to continue or stop the adjustment with reference only to the current realization of V_t. For example, if the "regulated" value $V_t - r_t$ goes below V^*, in the interval $[T^*, T^{*\prime})$ where $T^{*\prime} = \inf(t \geq T^* \mid V_t - V^* = 0^-)$, the firm may be willing to stop adjusting profits to increase its value. However, for the sake of perfectness, earlier interruption is not feasible as long as the threat of contract closure is credible. Credibility stems from the fact that the regulator's option to revoke if the firm deviates from r_t is always worth exercising at $V_t \geq V^*$, i.e. $F(V_t) \geq F(V^*) > 0$. At $T^{*\prime}$, however, the firm can set $r_{T'} = 0$ without revocation is carried out, and the game starts afresh. The sequence of moves in this game is shown in Figure 1.

Figure 1 - about here -

4 Discussion of the results

Although the resulting profit control mechanism is simple and proportional to the firm's value, several novel implications follow from our model. We can now summarize the discussions of our results in a series of subsections.

4.1 The properties of profit control

This profit control has several interesting properties:

- In (16) we can observe that the amount the firm is willing to share depends on the regulator's behaviour only through dt time units ago, which is interpreted as a reaction time. In other words, if the firm does not wish to reduce profits when $V_t \geq V^*$ it takes dt units of time for the regulator to analyze and react.[31]

- The optimal profit control r_t in (17) represents the cumulative amount of the project's value that the firm forgoes up to time t. The firm must increase r_t fast enough to keep $V_t - r_t$ below V^* but - subject to this constraint - wishes to adjust as little as possible.

- The control r_t is non-decreasing within $[T^*, T'^*)$;

- The control r_t is parametrized by the initial condition V^* which, in turn, depends on the revocation cost I and on the opportunity cost of direct management λ;

- An increase in I and λ decreases r_t;

- Finally, as r_t depends only on the primitive exogenous process V_t, the "regulated" process $V_t - r_t$ is also a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81).

The first two properties relate profit regulation to past realizations of V_t and then to the history of the contract. Since V_t fluctuates stochastically, although the intervention is continuous its rate of change is discontinuous. Yet, as for the discrete-time example, to avoid revocation the flow of payments must not be decreasing, and is smaller as I and λ are larger. Finally, the last property is important as it effectively makes the "regulated" process (16) a function solely of the starting state. At the beginning of each period both the firm and the regulator can predict the evolution of V_t by its current state only, which makes any sub-game beginning at a point at which revocation has not taken place equivalent to the whole game.

[31] See footnote 28.

4.2 Price adjustment

As is argued in the introduction and documented by Sappington (2002), most recent PCR plans for monopoly regulation do not simply cap prices, but also institute earnings ceiling whose violation triggers profit-sharing with customers. In practice, in the event of the firm's profits going beyond a "pre-determined" level, these plans require the x factor to be automatically adjusted upward, making the price cap adjustment rate $RPI - x$ more stringent.

What is the profit deadband that should trigger revision of the price cap mechanism? And what is the right revision level of the x factor to optimize expected welfare? Our model enables us to answer to these questions.

- First, the PS rule (17) arises endogenously as the optimal response from the continuous interaction between the firm and the regulator;
- Second, the rule is dynamic; the repetition of the relationship implicitly establishes the terms of a long-term contract, which guarantees the firm a "permitted" level of profits;
- Third, the optimal deadband is given by V^*: the firm's value is allowed to evolve according to geometric Brownian motion (7) until it hits V^*. At this point the price adjustment rule $RPI - x$ is revised to stop the process V_t from going any higher, and the Brownian motion describing the regulated profits is now given by (16), i.e.:

$$dV_t = (RPI - x')V_t dt + \sigma V_t dW_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \tag{19}$$

  where:

- $x' = x - \frac{d \inf_{0 \le v \le t} (V^*/V_v)/dt}{\inf_{0 \le v \le t} (V^*/V_v)} > x$ is the endogenous new price decrease factor.[32]

Let us now discuss the price adjustment behind the PS rule (17) in detail. If the numerical value for V^* is known, the optimal policy (11) can be

[32] Panteghini and Scarpa (2003) and Moretto et al. (2003) consider a similar problem in a continuous time stochastic model of investment. However, in their model the $RPI - x$ rule remains in place as long as profits are below an exogenously given level $\tilde{V}$, and, if $V_t > \tilde{V}$, the price decrease factor increases exogenously from x to x'.

written as $p_t\theta_t = \frac{\beta_1}{\beta_1-1}\frac{(\rho-\alpha)}{1-\lambda}(I-\hat{V})$, from which the boundary value for θ^* is determined by:

$$\theta^*(p_t) = \frac{\beta_1}{\beta_1-1}\frac{(\rho-\alpha)}{1-\lambda}\frac{I-\hat{V}}{p_t} \tag{20}$$

For any given value of the price cap p_t, random fluctuations of θ_t move the point (θ_t, p_t) horizontally left or right. If the point goes to the right of the boundary, then a price reduction is made immediately shifting the point down to the boundary. If θ_t stays to the left of the boundary, no new price regulation is made. Price reduction proceeds gradually to maintain (20) as an equality. For example, setting $RPI - x = 0$ so that $p_t = p_0$ for all t, by inverting (20) we obtain the optimal boundary function $p(\theta_t)$ which determines the optimal price regulation as a function of the sole state variable θ_t:

$$p_t = p_0\left(\frac{\theta^*}{\theta_t}\right) \qquad \text{with} \quad \frac{dp_t}{d\theta_t} < 0 \tag{21}$$

The boundary function for this case is shown in Figure 2.

Figure 2 - about here -

4.2.1 PCR vs. ROR

Many authors have argued that setting a PS rule in a PCR is a way of introducing some elements of rate-of-return regulation (ROR) in a regulatory plan (Crew and Kleindorfer, 1996; Sappington and Weisman, 1996; Weisman 1993, 2002; Sappington, 2002). Our framework helps us to clarify this argument. Simple algebra allows us to write (17) as a one-side sliding scale over an "allowed" rate-of-return:[33]

$$s_t^r = s_t + h_t\left(s^* - s_t\right), \qquad \text{with } h_t = \begin{cases} 0 & , \quad \text{for } s_0 \leq s_t < s^* \\ \frac{1-\inf\limits_{T^*\leq v\leq t}(V^*/V_v)}{1-(V^*/V_t)}, & \quad \text{for } s_t \geq s^* \end{cases} \tag{22}$$

where $s_t^r = \frac{V_t - r_t}{I}$, $s_t = \frac{V_t}{I}$ and $s^* = \frac{V^*}{I}$.

[33] Joskow and Schmalensee (1986, p. 29) have proposed a similar formula that would adjust prices so that the actual rate of return s_t^r at new prices would be given by: $s_t^r = s_t + h\left(s^* - s_t\right)$, where s_t is the rate of return at the prices in the year t (old prices), h is a constant between zero and one and s^* is the ROR target.

By (22), the actual rate of return under regulation s_t^r is given by the actual rate of return without regulation s_t, i.e. at the prices of time t, plus the adjustment $s^* - s_t$, where the rate at which contract s^* will be revoked, serves as the upper "allowed" rate of return. Thus, if at time t the rate of return goes above s^*, the price is adjusted according to (20) to reduce the rate of return by the fraction $h_t \geq 1$ of the difference between $s^* - s_t$. Unlike the formula proposed by Joskow and Schmalensee (1986), in (22) h_t is time-dependent and non-decreasing. That is, h_t is the optimal adjustment rate that keeps the regulator indifferent between revoking the contract and leaving the project to the firm. In this respect h_t cannot decrease when the difference $s^* - s_t$ narrows. In the period $0 \leq t < T^*$ where $s_t < s^*$, we will have $h_t = 0$ and $s_t^r = s_t$. During this regulatory lag the firm is allowed to earn the actual rate of return at the rates fixed at time $t = 0$ (which represents a period of "pure" PCR). When $s_t \geq s^*$, in period $t \geq T^*$, the adjustment rate h_t jumps to 1 and remains at that value until $dV_t > 0$ so that $s_t^r = s^*$. The firm is allowed to earn a rate of return no greater than the upper rate $s^* = \frac{\beta_1}{\beta_1 - 1} > 1$. That is, we get a period of ROR regulation. However, in periods where $dV_t < 0$ we will have $h_t > 1$ in order to keep the difference $s_t^r - s_t$ constant at the highest level reached up to t.

Given that in our simple model the regulator maintains ownership of the asset and there are no new investments, the cost I of revoking the contract is the only capital value (rate base) used by the regulator to set the maximum rate-of-return.[34] In other words, in valuing the invested capital as a basis for calculating the "reasonable" market return the firm can earn, the regulator should also include his cost of "exercising the revocation option" as a way of taking account of the difficulties of inducing the firm to accept future revision of contractual terms.

4.3 Regulatory lags and the commitment problem

Although formally most regulatory contracts specify both the duration and the review periods, in reality it is more and more common for the regulator to change the firm's rates unilaterally and/or move up revisions. The best known example is Oftel, which raised the x-factor in a PC plan for British

[34] If revocation requires some contractual indemnities (i.e. the underappreciated value of investment in technology and infrastructures) to the firm, with $-V + Ind. < 0$, we can set $I = Ind. + I'$ and $I' \geq 0$.

Telecom well before the scheduled review date.[35] In our model the regulator's ability to ratchet the x-factor upward is related to the threat of revocation. Therefore, the strength of this threat determines the difference between the contractual and actual duration of a regulatory arrangement, and hence, the success of introducing a PS rule in the original PCR.

The difference between contractual and real contract duration is another way of looking at the well-known "commitment problem" in regulation. Crew and Kleindorfer (1996) argue that a major issue in incentive regulation is commitment: "If a company is concerned that the regulator will penalize it at the end of or even during the price-cap period if it is successful, it may not pursue efficiency as strongly as implied by the apparent incentives of PCR. Thus, the notion that the regulator will not renege on the terms of PCR is very important for efficiency to be achieved....(p.218)". However, they subsequently admit that as the regulators' goal is rent extraction, it is easy to see that they have limited incentives to commit, and that this problem is at the root of the emergence of regulatory contracts thar incorporate sharing rules.[36]

Besides the trade-off between commitment and reneging raised by Crew and Kleindorfer (1996) and others[37], we have seen that the credibility of the revocation option becomes relevant to the renegotiation process itself, since it determines the regulator's bargaining power and thus the timing of contract renewal.[38] If the cost of revocation measures the "inefficiencies" that the regulator incurs in direct provision of the utility or in trying to find a new franchisee, or through political and litigation costs, this cost also raises the problem of the irreversibility of the contract.[39] For example, in the case of

[35] See Amstrong at al. (1994) and Sappington and Weisman (1996) for other examples.

[36] "Such devices provide sharing of gains to ratepayers and therefore might be seen to be less vulnerable to reneging by the regulator if the company does well. In addition, such devices, in limiting how well the company can do, make the regulator less likely to renege" (Crew and Kleindorfer,1996, p.218).

[37] See also Breautingam and Panzar (1989) and Weisman (1993).

[38] The result of an endogenous regulatory lag may also explain the empirical evidence that while contracts between regulators and private firms may be of limited duration, they are often renewed without any variation in contractual terms (Joskow and Schmalensee, 1986, p.7).

[39] In this case, taxation of excess profits may be a socially less expensive indirect way of diverting part of the firm's profits to the consumers.

local utilities, after the contract is signed it is the municipal authority that plays the role of a regulator with respect to the private firm. The inexperience of the municipal authority in this role may damage its credibility and thus determine a negotiating disadvantage (Clark and Mondello, 2000).

5 Concluding Remarks

Recent British and US experience records increasingly frequent intervention by regulators to induce public utlilities firms to accept some form of profit-sharing; that is, a procedure for distributing to consumers a portion of the firm's profits in the form of lower prices.

Following these experiences, in this paper we have derived an optimal profit-sharing (PS) scheme as a unilateral decision by the regulator to favour the firm's customers. In our setting, if the regulator observes that the firm is making excessively high profits, he imposes a sharing rule and if the firm does not agree to it, he revokes the contract.

We model revocation as an outside option of the regulator, in which the value of the firm counts along with social welfare and cost of revocation to the regulator (Proposition 1). Essentially, we find that the threat of revocation determines an endogenous and non-decreasing state-contingent proportional rule by which the firm "controls" its profits, and that this is a sub-game perfect equilibrium (Proposition 2). However, as the threat of revocation can be very costly, the long-run equilibrium may show quite a considerable regulatory lag.

In closing, let us briefly suggest a potential extension of our analysis. The economic literature on profit-sharing regulation generally holds that sharing rules diminish the firm's incentive to invest (Lyon, 1996; Crew and Kleindorfer, 1996; Sappington, 2002; Moretto et al., 2003). As noted in Section 5, our model does not take investment into account. It could be interesting to allow for investment during the regulatory period and assess the effects of the threat of revocation on the firm's investment decisions.

A Appendix

A.1 Proof of Proposition 1

Using standard results on expected present value with barriers (Harrison, 1985), it easy to write (9), for any arbitrary investment trigger V^*, as:

$$\begin{aligned} F(V) &= \left[(1-\lambda)V^* - (I-\hat{V})\right] E_0\left[e^{-\rho T} \mid V_0 = V\right] \\ &= \left[(1-\lambda)V^* - (I-\hat{V})\right] \left(\frac{V}{V^*}\right)^{\beta_1} \end{aligned} \tag{23}$$

Then maximizing (23) with respect to V^* requires:

$$\frac{dF}{dV^*} = \left[(1-\beta)(1-\lambda) + \beta\frac{I-\hat{V}}{V^*}\right]\left(\frac{V}{V^*}\right)^{\beta_1} = 0,$$

from which we obtain (11):

$$V^* = \frac{\beta_1}{\beta_1 - 1}\frac{1}{1-\lambda}(I-\hat{V}) \tag{24}$$

Easy calculation also shows that the second order condition is satisfied:

$$\frac{d^2F}{d(V^*)^2} = -\left(\frac{V}{V^*}\right)^{\beta_1}\beta\frac{I-\hat{V}}{(V^*)^2} < 0$$

Finally, substituting (24) into (23) and rearranging, we may rewrite the latter in the form

$$F(V) = A(V^*)V^{\beta_1},$$

where:

$$A(V^*) \equiv \left[(1-\lambda)V^* - (I-\hat{V})\right] V^{*-\beta_1} \equiv \frac{1-\lambda}{\beta_1}(V^*)^{1-\beta_1} > 0. \tag{25}$$

Note that the same result can be obtained by contingent claims analysis (Cox and Ross, 1976; Harrison and Kreps, 1979). By arbitrage arguments and applying Ito's lemma to F, the value of the regulator's option to revoke (i.e. his outside option) is given by solution of the following differential equation (Dixit and Pindyck, 1994, p. 147-152):

$$\frac{1}{2}\sigma^2V^2F'' + \alpha VF' - \rho F = 0 \quad \text{for } V \in (0, V^*], \tag{26}$$

where $F(V)$ must satisfy the following boundary conditions:

$$\lim_{V \to 0} F(V) = 0 \tag{27}$$

$$F(V^*) = (1 - \lambda)V^* - (I - \hat{V}) \tag{28}$$

$$F(V^*) = 1 - \lambda \tag{29}$$

If the value of the utility goes to zero, so does the option to revoke. Conditions (28) and (29) for efficient operation imply respectively that, at the trigger level V^*, the value of the option is equal to its liabilities where I indicates the sunk cost of revocation (*matching value condition*) and suboptimal exercise of the option is ruled out (*smooth pasting condition).* By the linearity of (26) and using (27), the general solution is of the form:

$$F(V) = AV^{\beta_1}, \tag{30}$$

where A is a constant to be determined and $\beta_1 > 1$ is the positive root of the quadratic equation:

$$\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0 \tag{31}$$

As (30) represents the option value of optimally revoking the contract, the constant A must be positive and the solution is valid over the range of V for which it is optimal for the regulator to keep the option alive $(0, V^*]$. By substituting (30) for (28) and (29) we get (11) (i.e. (24)) and (12) (i.e. (25)).

A.2 Proof of Propositions 2 and 3

We prove that the regulatory scheme proposed is a perfect equilibrium belonging to the potentially very large class of efficient perfect equilibria in pure strategies for the continuous time threat-game described in above Section 4. We proceed in the following way. First, we define the profit-sharing scheme as a one-sided non-decreasing control (as in Harrison, 1985) on the state variable V. Then we prove that when the firm controls its profits by this mechanism, the regulator's option to revoke the contract is always equal to zero, which makes the sharing scheme a good candidate for long-run equilibrium in threat strategies. Next, we prove that this is indeed the case by showing that any deviation from the proposed scheme makes revocation worthwhile. The non-decreasing property of the proposed scheme is crucial to this result. Finally, since V is a Markov process in levels, it is easy to ascertain that the equilibrium is sub-game perfect.

1) Regulation mechanism

We define the regulation as the reduction dV_t needed to keep V_t at V^*. That is, the policy control consists of a process $Z = \{Z_t, t \geq 0\}$ and a regulated process $V^r = \{V_t^r, t \geq 0\}$ such that

$$V_t^r \equiv V_t Z_t, \quad \text{for} \quad V_t^r \in (0, V^*], \tag{32}$$

where:

- $i)$ V_t is a geometric Brownian motion, with stochastic differential as in (7);
- $ii)$ Z_t is a decreasing and continuous process with respect to V_t ;
- $iii)$ $Z_0 = 1$ if $V_0 \leq V^*$, and $Z_0 = V^*/V_0$ if $V_0 > V^*$ so that $V_0^r = V^*$;
- $iv)$ Z_t decreases only when $V_t^r = V^*$.

Applying Ito's lemma to (32), we get:

$$d\,V_t^r = \alpha\, V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}, \qquad V_0^r \in (0, V^*]$$

where $V_t^r \frac{dZ_t}{Z_t} \equiv V_t dZ_t = -dr_t$ is the infinitesimal level of value forgone up by the firm. In terms of the regulated process V_t^r, we can write:

$$r_t \equiv r(V_t) = V_t - V_t^r \equiv (1 - Z_t) V_t, \tag{33}$$

Although the process Z_t may have a jump at time $t = 0$ it is continuous and keeps V_t below the barrier using the minimum amount of control, in that control is exercised only when V_t crosses V^* from below with probability one in the absence of regulation. Therefore, in the case of $V_0 < V^*$, we get $V_t^r \equiv V_t$, with initial condition $V_0^r \equiv V_0 = V$, and $Z_t = 1$. At $T^* \equiv T(V^*) =$

$\inf(t \geq 0 \mid V_t - V^* = 0^+)$ control starts so as to maintain $V_t^r = V^*$. The firm adjusts the project's value downward by the amount $r_t = V_t - V_t^r \geq 0$ every time V^* is hit. The same conditions $(i) - (iv)$ uniquely determine Z_t with the representation form (Harrison,1985; Proposition 3, p. 19-20):[40]

$$Z_t \equiv \begin{cases} \min(1, V^*/V_0) & \text{for } t = 0 \\ \inf\limits_{0 \leq v \leq t} (V^*/V_v) & \text{for } t \geq 0 \end{cases} \tag{34}$$

Figure 3 -about here-

2) The value of revocation

Although the price adjustment reduces the value of the project but keeps the contract alive, the firm always prefers to stop the adjustment if the threat of revocation is not carried out, i.e. $r_t = V_t - V_t^r \geq 0$, for all $t \geq T^*$. The regulator is not in the same condition. Indicating by $F^r(V; V^*)$ the regulator's value of the option when the firm controls its profits, this can be expressed, at time zero, by (to simplify discussion, we assume that $V_0 < V^*$ so that $T^* > 0$):

$$F^r(V; V^*) = \max E_0 \left\{ ((1-\lambda)V_T^r - (I - \hat{V}))e^{-\rho T} \mid V_0 = V \right\} \tag{35}$$

or using $r_t = V_t - V_t^r = (1 - Z_t)V_t$:

$$F^r(V; V^*) = \min E_0[((1-\lambda)V_T - (I - \hat{V}))e^{-\rho T} + (1-\lambda)(V_T^r - V_T)e^{-\rho T} \mid V_0 = V] \tag{36}$$

In (36) the option value, with a barrier control on V_t, takes account of two terms depending upon the joint evolution of V_t and V_t^r. The first $(1-\lambda)V_T - (I - \hat{V})$ is the "social welfare" without the barrier (i.e. the value of the firm

[40] This is an application of a well-known result of Levy (1948), for which the process:

$$\ln V_t^r \equiv \ln V_t + \ln Z_t \equiv \ln V_t - \inf_{0 \leq v \leq t} (\ln V_v - \ln V^*)$$

has the same distribution as the "reflected Brownian process" $\mid \ln V_t - \ln V^* \mid$.

net of cost of revocation and social losses), while $(1-\lambda)(V_T^r - V_T)$ is the increase in welfare due to the profit adjustment. Keeping the dependence of F^r on V_t^r active and assuming it is twice continuously differentiable, by Ito's lemma we obtain:

$$dF^r(V_t^r;V^*) = \frac{1}{2}F^{r''}V_t^{r2}\sigma^2 dt + F^{r'}\alpha\, V_t^r dt + F' V_t^r dW_t + F^{r'}V_t^r \frac{dZ_t}{Z_t} \tag{37}$$

As $dZ_t = 0$ except when $V_t^r = V^*$ becomes:

$$dF^r(V_t^r;V^*) = [\frac{1}{2}\sigma^2 V_t^{r2} F^{r\prime\prime}(V_t^r;V^*) + \alpha\, V_t^r F^{r\prime}(V_t^r;V^*)]dt \tag{38}$$

$$+\sigma V_t^r F^{r\prime}(V_t^r;V^*)\, dW_t - F^{r\prime}(V^*;V^*) dr(V_t)$$

Integrating by part the process $F^r e^{-\rho T^*}$ gives:

$$e^{-\rho T^*}F^r(V_T^r;V^*) = F^r(V;V^*) + \tag{39}$$

$$+\int_0^{T^*} e^{-\rho s}\left[\frac{1}{2}\sigma^2 V_s^{r2} F^{r\prime\prime}(V_s^r;V^*) + \alpha\, V_s^r F^{r\prime}(V_s^r;V^*) - \rho F^r(V_s^r;V^*)\right] ds$$

$$+\sigma \int_0^{T^*} e^{-\rho s} V_s^r F^{r\prime}(V_s^r;V^*)\, dW_s - F^{r\prime}(V^*;V^*)\int_0^{T^*} e^{-\rho s} dr(V_s)$$

Taking the expected value of (39), if the following conditions apply:

(a) $e^{-\rho t}V_t^r F^{r\prime}(V_t^r;V^*)$ is bounded within $(0, V^*]$

(b) $F^r(V_{T^*}^r;V^*) = (1-\lambda)V_T^r - (I - \hat{V})$

(c) $F^{r\prime}(V^*;V^*) = 0$;

(d) $\frac{1}{2}\sigma^2 V_t^{r2} F^{r\prime\prime}(V_t^r;V^*) + \alpha V_t^r F^{r\prime}(V_t^r;V^*) - \rho F^r(V_t^r;V^*) = 0$

we obtain the expression for $F^r(V;V^*)$ as in (35). Now the two conditions (b) and (c) together with the fact that at T^* the adjustment starts so as to keep $V_t^r = V^*$ (i.e. compare condition (c) with condition (29)), give $F^r(V; V^*) = 0$.

A heuristic but direct way of looking at the same result is to see $F^r(V; V^*)$ as the sum of the value of the regulator's revocation option (net of welfare loss) $F(V) = A(V^*)V^{\beta_1}$ and the expected value to the firm of future cumulative controls due to the adjustment weighted for the distributional factor $1 - \lambda$. Let us then denote by $R(V^r; V^*)$ the expected value of cumulative future losses in the firm's value due to the adjustments. The rational player evaluates R considering an infinite-life project:

$$\begin{aligned} R(V_0^r; V^*) &= E_0 \left\{ \int_0^\infty e^{-\rho t} dr(V_t) \mid V_0^r \in (0, V^*] \right\} \\ &= -E_0 \left\{ \int_0^\infty e^{-\rho t} V_t dZ_t] \mid V_0^r \in (0, V^*] \right\} \end{aligned} \tag{40}$$

Since V_t^r is a Markov process in levels (Harrison, 1985, proposition 7, p.80-81), we know that the foregoing conditional expectation is in fact a function of the starting state alone.[41] Again, keeping the dependence of R on V_t^r active and assuming that it is twice continuously differentiable, by Ito's lemma we get:

$$\begin{aligned} dR &= R' dV_t^r + \frac{1}{2} R''(dV_t^r)^2 \\ &= R'(Z_t dV_t + V_t dZ_t) + \frac{1}{2} R'' Z_t^2 (dV_t)^2 \\ &= R'(\alpha V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}) + \frac{1}{2} R'' Z_t^2 \sigma^2 dt \\ &= \frac{1}{2} R'' \sigma^2 V_t^{r2} dt + R' \alpha V_t^r dt + R' \sigma V_t^r dW_t + R' V_t^r \frac{dZ_t}{Z_t} \end{aligned} \tag{41}$$

[41] For $V_0 = V > V^*$ optimal control would require Z to have a jump at zero so as to ensure $V_0^r = V^*$. In this case the integral on the right of (40) is defined to include the control cost r_0 incurred at $t = 0$, that is (see Harrison 1985, p.102-103):

$$\int_0^\infty e^{-\rho t} dr_t \equiv r_0 + \int_{(0,\infty)} e^{-\rho t} dr_t$$

where $r_0 = V - V_0^r$.

where it has been taken into account that for a finite-variation process like Z_t, $(dZ_t)^2 = 0$. As $dZ_t = 0$ except when $V_t^r = V^*$ we are able to rewrite (41) as:

$$dR(V_t^r; V^*) = [\frac{1}{2}\sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha\, V_t^r R'(V_t^r; V^*)]dt \tag{42}$$

$$+\sigma V_t^r R'(V_t^r; V^*)\, dW_t - R'(V^*; V^*) dr(V_t)$$

This is a stochastic differential equation in R. Integrating by part the process Re^{-rt} we get (Harrison, 1985, p.73):

$$e^{-\rho t} R(V_t^r; V^*) = R(V_0^r; V^*) + \tag{43}$$

$$+ \int_0^t e^{-\rho s} \left[\frac{1}{2}\sigma^2 V_s^{r2} R''(V_s^r; V^*) + \alpha\, V_s^r R'(V_s^r; V^*) - \rho R(V_s^r; V^*)\right] ds$$

$$+\sigma \int_0^t e^{-\rho s} V_s^r R'(V_s^r; V^*)\, dW_s - R'(V^*; V^*) \int_0^t e^{-\rho s} dr(V_s)$$

Taking the expectation of (43) and letting $t \to \infty$, if the following conditions apply:

(a) $\lim_{l \to 0} \Pr[T(l) < T(V^*) \mid V_0^r \in (0, V^*]] = 0$ for $l \leq V_t^r < V^* < \infty$, where $T(l) = \inf(t \geq 0 \mid V_t^r = l)$ and $T(V^*) = \inf(t \geq 0 \mid V_t^r = V^*)$;

(b) $R(V_t^r; V^*))$ is bounded within $(0, V^*]$;

(c) $e^{-\rho t} V_t^r R'(V_t^r; V^*)$ is bounded within $(0, V^*]$;

(d) $R'(V^*; V^*) = 1$;

(e) $\frac{1}{2}\sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha\, V_t^r R'(V_t^r; V^*) - \rho R(V_t^r; V^*) = 0$,

we obtain $R(V^r; V^*)$ as indicated in (40). Condition (a) says that the probability of the regulated process V_t^r reaching zero before reaching another point within the set $(0, V^*]$ is zero. As V_t^r is a geometric type of process this condition is, in general, always satisfied (Karlin and Taylor, 1981, p. 228-230). Furthermore, if condition (a) holds and $R(V^r; V^*)$ is bounded, then conditions (b) and (c) also hold. According to the linearity of (e) and using (d), the general solution has the form:

$$R(V_0^r; V^*) = B(V^*)(V_0^r)^{\beta_1}, \tag{44}$$

with:

$$B(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0. \tag{45}$$

As for $V_0 \leq V^*$, $Z_0 = 1$ and $V_0^r = V_0 = V$, then $R(V_0^r; V^*) = R(V; V^*)$. On the other hand, if $V_0 > V^*$, we get $Z_0 = V^*/V_0$, so that $V_0^r = V^*$ and $R(V_0^r; V^*) = R(V^*; V^*)$.

Finally, putting together (30) and (44) we get:

$$F^r(V_t; V^*) = A(V^*)V_t^{\beta_1} - (1-\lambda)B(V^*)V_t^{\beta_1} = 0,$$

i.e. the revocation option goes to zero if the value of the welfare losses is exactly offset by the benefits from adjustment of the firm (lower profits). For $0 < t < T^*$, $V_t^r \equiv V_t$ and then $F^r(V_t) \equiv F(V_t)$. At T^*adjustment starts, killing the option, i.e. $F^r(V) = 0$, for all $t \geq T^*$.

3) Optimal revocation strategy and perfect equilibrium

Since V_t follows a random walk there is, for each small time interval of length dt, a constant probability that the game will continue one more period. The game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite. Neither player can perfectly predict V_t at any date and the adjustment scheme described by (33) with form (34) is viewed by both agents as a stationary strategy for evaluating all future value reductions.[42] In the strategy space of the regulator it appears as:

[42] It is well known that infinitely repeated games may be equivalent to repeated games that terminate in finite time. At each period there is a probability that the game continues one more period. The key is that the conditional probability of continuing must be positive (Fudenberg and Tirole, 1991, p.148). Integrating the differential form (7), the geometric Brownian motion can be expressed as:

$$V_{t+dt} = V_t e^{dY_t}$$

where $dY_t = \mu dt + \sigma dW_t$ and $\mu = \alpha - \frac{1}{2}\sigma^2$. The differential dY_t is derived as the continuous limit of a discrete-time random walk, where in each small time interval of length Δt the variable y either moves up or down by Δh with probabilities (Cox and Miller, 1965, p.

$$\phi(V_t, r_t) = \begin{cases} \text{Do not revoke at } t = T^* \text{ if the firm} \\ \text{plays the rule } r_t = (1 - Z_t)V_t \text{ for } t' < t \\ \\ \text{Revoke if the firm deviates from} \\ r_t = (1 - Z_t)V_t \;\; \text{at any } t' < t \end{cases}$$

where $\phi(V_t, r_t)$ is the strategy at t with history (V_t, Z_t). The regulator's "threat" strategy is chosen if the firm deviates by adjusting V_t less than r_t or by abandoning $r_t = (1 - Z_t)V_t$ as a rule to evaluate future adjustments. The regulator must believe that the regulation, from the initial date and state (T^*, V^*), will be kept in use for the whole (stochastic) planning horizon. If the firm deviates, the regulator believes that the firm intends to switch to a different rule in the future and knows for sure that the regulator will revoke immediately thereafter. The regulator does not revoke at t if $r_{t'} \geq V_{t'} - V_{t'}^r$ for all $t' \leq t$, because value controls are expected to continue with the same rule and $F^r(V) = 0$ for all $t \geq T^*$. If $r_{t'} < V_{t'} - V_{t'}^r$ for some $t' < t$ the regulator expects a different rule and carries out the threat, switching from $F^r(V_t) = 0$ to $F(V_t) \geq V^* - I$. The game is over.

205-206):

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\sqrt{\Delta t}}{\sigma}\right), \quad \Pr(\Delta Y = -\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\sqrt{\Delta t}}{\sigma}\right)$$

or defining $\Delta h = \sigma\sqrt{\Delta t}$:

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\Delta h}{\sigma^2}\right), \quad \Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\Delta h}{\sigma^2}\right)$$

That is, for small Δt, Δh is of order of magnitude $O(\sqrt{\Delta t})$ and both probabilities become $\frac{1}{2} + O(\sqrt{\Delta t})$, i.e. not very different from $\frac{1}{2}$. Furthermore, considering again the discrete-time approximation of the process Y_t, starting at $V^* e^{+\Delta h}$, the conditional probability of reaching V^* is given by (Cox and Miller, 1965, ch.2):

$$\Pr(Y_t = 0 \mid Y_t = 0 + \Delta h) = \begin{cases} 1 & \text{if } \mu \leq 0 \\ e^{-2\mu\Delta h/\sigma^2} & \text{if } \mu > 0 \end{cases}$$

which converges to one as Δh tends to zero.

To prove this, first consider R as in (40). For each $t' > T^*$, integration by parts gives:

$$\int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = \tag{46}$$

$$e^{-\rho(t-t')} V_t Z_t - V_{t'} Z_{t'} + \rho \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds - \int_{t'}^{t} e^{-\rho(s-t')} Z_s dV_s$$

Taking expectation of both sides and using the zero-expectation property of the Brownian motion (Harrison, 1985, p.62-63), we have:

$$E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = E_{t'}[V_t Z_t e^{-\rho(t-t')}] - V_{t'} Z_{t'} + (\rho - \alpha) E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds \tag{47}$$

By the Strong Markov property of V_t^r[43], it follows that $E_{t'}[V_t Z_t e^{-\rho(t-t')}] = E_{t'}[V_t Z_t] E_{t'}[e^{-\rho(t-t')}] = V^* E_{t'}[e^{-\rho(t-t')}] \to 0$ almost as surely as $t \to \infty$, so that:

$$E_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} V_s dZ_s = -V_{t'} Z_{t'} + (\rho - \alpha) E_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} (V_s - r_s) ds$$

Since $-V_{t'} Z_{t'} + (\rho - \alpha) E_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} V_s ds = 0$, substituting (40) and rearranging we get:

$$R(V_{t'}; V^*) = (\rho - \alpha) E_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')} r_s ds \tag{48}$$

Secondly, let us assume (t', t) is an interval in which r_s is flat so that $V_s^r \leq V^*$, and t is the first time in which $dZ_t > 0$. Considering the decomposition (47) we can write (48) as:

$$\begin{aligned} R(V_{t'}; V^*) &= (\rho - \alpha) \left\{ E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'} \left\{ \int_{t}^{\infty} e^{-\rho(s-t')} r_s ds \right\} \right\} \\ &= (\rho - \alpha) \left\{ E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'} \left\{ e^{-\rho(t-t')} \int_{t'}^{\infty} e^{-\rho(s-t')} r_s^* ds \right\} \right\} \end{aligned}$$

[43] The Strong Markov Property of regulated Brownian motion processes stresses the fact that the stochastic first passage time t and the stochastic process V_t^r are independent (Harrison, 1985, Proposition 7, p.80-81).

where we have defined $V_s^{r*} = V_{t+s}^r$ and $r_s^* = r_{t+s} - r_t$ for $t' \leq t$. Applying, again, the Strong Markov Property of V_t^r we get:

$$\begin{aligned} R(V_{t'}; V^*) &= E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'} \left\{ e^{-\rho(t-t')} E_{t'} \int_{t'}^{\infty} e^{-\rho(s-t')\infty} r_s^* ds \right\} \\ &= (\rho - \alpha) E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'} \left\{ e^{-\rho(t-t')} R(V_{t'}; V^*) \right\} \\ &= (\rho - \alpha) E_{t'} \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + R(V_{t'}; V^*) E_{t'} \left\{ e^{-\rho(t-t')} \right\} \end{aligned}$$

Since $r_s = r_{t'} \equiv V_{t'} - V_{t'}^r$ for all $s \in (t', t)$ we can simplify the above expression as:

$$R(V_{t'}; V^*) = \frac{(\rho - \alpha)}{\rho} r_{t'} = \frac{(\rho - \alpha)}{\rho} (V_{t'} - V_{t'}^r) \tag{49}$$

>From (49), any application of controls $r_{t'} < V_{t'} - V_{t'}^r$, leads to a reduction of (48) for all $t \geq t'$ and then to $F^r(V_t; V^*) > 0$. Furthermore, the firm does not adjust by more than r_t since doing so would does not increase the probability of delaying revocation. It does not pay less, since $r_t < V_t - V_t^r$ induces closure making it worse off, i.e. $0 < V_t$.

Finally, as V_t^r is a Markov process in levels, it is immediate by (48) that any sub-game that begins at a point at which revocation has not taken place is equivalent to the whole game. The strategy ϕ is efficient for any sub-game starting at an intermediate date and state (t, V_t). We have sub-game perfection.

4) Non-decreasing path of r_t within $[T^*, T'^*)$.

So far we have implicitly assumed that, once started at T^*, the adjustment goes on forever. Earlier interruptions are not feasible as long as the threat of revocation is credible. Credibility relies on the fact that the agency's option to revoke if the firm deviates from r_t is always worth exercising at $V_t > V^*$, i.e. $F(V_t) \geq F(V^*) > 0$. As the decision rule strategy depends on the history of the game, the regulator expects price adjustment to continue according to the rule r_t and any premature halt could make it no longer subgame-perfect. However, in an optimal Brownian path there is a positive probability of the primitive process V_t crossing V^* again starting at an interior point of the range (V^*, ∞). In this case, the firm may be willing to stop adjusting the price. That is, the firm controls its value until $V_t \geq V^*$, letting the agency expect adjustment to continue according to the same rule $r_t = (1 - Z_t)V_t$, but when V_t reaches, for the first time after T^*, a predetermined level, say $V' \leq V^*$, it ceases the adjustment. The regulator will face a jump from zero to $F(V') \leq F(V^*)$ making the threat of revocation no longer credible. To see this, consider the possibility of the firm's adjustment terminating at time T' with $T^* < T' < \infty$, where $T' = \inf(t \geq T^* \mid V_t \geq V')$ is the first time that V' $\leq V^*$ when regulation adjustment is under way. The value of the revocation option starting at any $t \in [T^*, \infty)$ can be expressed as:

$$\tilde{F}^r(V_t; V') = P(V'; V_t)E_t[F^r(V_{T'})e^{-r(t-T')}] + \tag{50}$$

$$(1 - P(V'; V_t)) \max E_t[(V_T^r - I)e^{-r(t-T)}]$$

where $P(V'; V_t)$ is the probability of the unadjusted process V_t reaching $V' \leq V^*$ starting at an interior point of the range (V^*, ∞), which is equal to (Cox and Miller, 1965, p. 232-234):

$$\Pr(T' < \infty \mid V_t) \equiv P(V'; V_t) = \left(\frac{V_t}{V'}\right)^{-2\mu/\sigma^2}$$

with $\mu = (\alpha - \frac{1}{2}\sigma^2)$.[44] As the starting point is now any $t \in (T^*, \infty)$, we can immediately see in (50) the dependence on both V_t^r and V_t. Since the option value in the case of regulation is zero, and at time T' when the contract is recalled it is simply $F^r(V_{T'}) = F^r(V')$, we get:

$$\tilde{F}^r(V_t; V') = P(V'; V_t)E_t[F^r(V')e^{-r(T'-t)}]$$

According to the Strong Markov Property of V_t^r equation (50) becomes:

$$\tilde{F}^r(V_t; V') = P(V'; V_t)F^r(V')\left(\frac{V_t}{V'}\right)^{\beta_2} \tag{51}$$

where $\beta_2 < 0$ is the negative root of (31). Since at t the unregulated process V_t is greater than V' and $P(V'; V_t)\left(\frac{V_t}{V'}\right)^{\beta_2} = \left(\frac{V_t}{V'}\right)^{\beta_2 - 2\mu/\sigma^2} \leq 1$, we obtain $\tilde{F}^r(V_t; V') \leq F^r(V')$ for all $t \in [T^*, T')$, which implies that:

[44] This probability is $P(V'; V_t) = 1$ for $\mu \leq 0$, see footnote no. 42.

$$\tilde{F}^r(V_t; V') = F^r(V^*) \left(\frac{V'}{V^*}\right)^{\beta_1} \left(\frac{V_t}{V'}\right)^{\beta_2 - 2\mu/\sigma^2} \leq F^r(V^*) \tag{52}$$

Therefore, to prevent the regulator revoking the contract, the price adjustment continues until time $T'^* \equiv T'(V^*) = \inf(t \geq T^* \mid V_t - V^* = 0^-)$ when the trigger value V^* is hit again for the first time after T^*. The game ends and can then be started afresh.

References

[1] Armstrong, M., Cowan, S. and J. Vickers (1994), *Regulatory Reforms: Economic Analysis an British Experience,* The MIT Press: Cambridge.

[2] Armstrong, M., Cowan, S. and J. Vickers (1995), "Nonlinear pricing and price cap regulation", *Journal of Public Economics*, **58**, 33-55.

[3] Bergin, J., and W.B. MacLeod, (1993), "Continuous Time Repeated Games", *International Economic Review,* **34**, 21-37.

[4] Blackmon, G. (1994), *Incentive regulation and the regulation of incentives*, Kluver Academic Press: Boston.

[5] Braeutigam, R. and J. Panzar, (1989), "Diversification incentives under 'price-based' and 'cost-based' regulation", *Rand Journal of Economics,* **20**, 194-198.

[6] Brennan, T. and J. Boyd, (1997), "Stranded Costs, Takings, and the Law and Economics of Implicit Contracts", *Journal of Regulatory Economics*, **11,** 41-54.

[7] Clark E., and G. Mondello, (2000): "Resource Management and the Mayor's Guarantee in French Water Allocation", *Environmental and Resource Economics*, **15**, 103-113.

[8] Cox, D.R. and H.D. Miller, (1965), *The Theory of Stochastic Process,* Chapman and Hall: London.

[9] Cox, J.C. and S.A. Ross, (1976), "The Valuation of Options for Alternative Stochastic Process", *Econometrica*, **53**, 385-408.

[10] Chennells, L., (1997), "The windfall tax", *Fiscal Studies*, **18**, 279-291.

[11] Crew, M. and P. Kleindorfer, (1996), "Incentive Regulation in the United Kingdom and United States: some lessons", *Journal of Regulatory Economics*, **9**, 211-225.

[12] Demski, J.S. and D.E.M. Sappington (1991), "Resolving Double Hazard Problem with Buyout Agreements", *RAND Journal of Economics*, **22**, 232-240.

[13] Dixit, A., and R. Pindyck, (1994), *Investment under Uncertainty,* Princeton University Press: Princeton.

[14] Fudenberg, D., and J. Tirole, (1991), *Game Theory*, The MIT Press: Cambridge MA.

[15] Goldberg, V.P., (1976), "Regulation and administered contract", *Bell Journal of Economics*, **7**, 426-448.

[16] Harrison, J.M., and D. Kreps, (1979), "Martingales and Arbitrage in Multiperiod Securities Markets", *Journal of Economic Theory*, **20**, 381-408.

[17] Harrison, J.M., and M.T. Taksar, (1983), "Instantaneous Control of Brownian Motion", *Mathematics and Operations Research,* **8**, 439-453.

[18] Harrison J.M., (1985), *Brownian Motions and Stochastic Flow Systems,* Wiley: New York.

[19] Karlin, S. and H.M. Taylor, (1981), *A Second Course in Stochastic Processes,* Academic Press: New York.

[20] Klein, D. and B.O. Flaherty, (1993), "A game Theoretic Rendering of Promises and Threats", *Journal of Economic Behavior and Organization*, **21,** 295-314.

[21] Joskow, P. and R. Schmalensee, (1986), "Incentive Regulation for Electric Utilities", *Yale Journal on Regulation*, **4**, 1-49.

[22] Laffont J.J. and J. Tirole, (1994): *A Theory of Incentives in Procurement and Regulation*, The MIT Press, Cambridge, Massachussetts.

[23] Levy P., (1948), *Processus Stochastiques et Mouvement Brownien,* Gauthier-Villars: Paris.

[24] Lyon T.P., (1996), "A Model of Sliding-Scale Regulation", *Journal of Regulatory Economics,* **9,** 227-247.

[25] McDonald, R. and D. Siegel, (1986), "The Value of Waiting to Invest", *Quarterly Journal of Economics,* **101,** 707-727.

[26] Moretto, M. and G. Rossini, (2003), "Designing Severance Payments and Decision Rights for Efficient Plant Closure under Profit-Sharing", in M.R. Sertel e S. Koray (eds.), *Adavances in Economic Design*, Springer-Verlag, Berlin, 363-386.

[27] Moretto, M., Panteghini P. and C. Scarpa, (2003), "Investment size and firm's value under profit-sharing regulation", CESifo Working Paper 1040.

[28] Moretto, M. and P. Valbonesi, (2000), "Option to revoke and regulation of local utilities", Nota di Lavoro n. 51, FEEM, Milan.

[29] Panteghini, P. and C. Scarpa, (2001), "Incentives to (Irreversible) Investments under Different Regulatory Regimes", *International Tax and Public Finance,* ***10****, 403-18.*

[30] Sappington D., (2002): "Price Regulation and Incentives", in M. Cave, Majumdar S. and I. Vogelsang (eds.), *Handbook of Telecommunications,* North-Holland, London.

[31] Sappington ,D. and D. Sibley (1992), "Strategic nonlinear pricing under price cap regulation", *RAND Journal of Economics*, **23**, 1-19.

[32] Sappington, D. and D. Weisman (1996), "Revenue sharing in incentive regulation plans", *Information Economics and Policy,* **8**, 229-248.

[33] Simon, L. and M. Stinchcombe, (1989), "Extensive Form Games in Continuous Time: Pure Strategies", *Econometrica*, **57**, 1171-1214.

[34] Shavell, S. and K. Spier, (1996), "Threats without Binding Commitments", NBER Working Paper 5461.

[35] Weisman, D.L., (1993), "Superior regulatory regimes in theory and in practice", *Journal of Regulatory Economics,* **5**, 355-66.

[36] Weisman, D.L., (2002), "Is there "hope" for price cap regulation", *Information Economics and Policy,* **14**, 349-70.

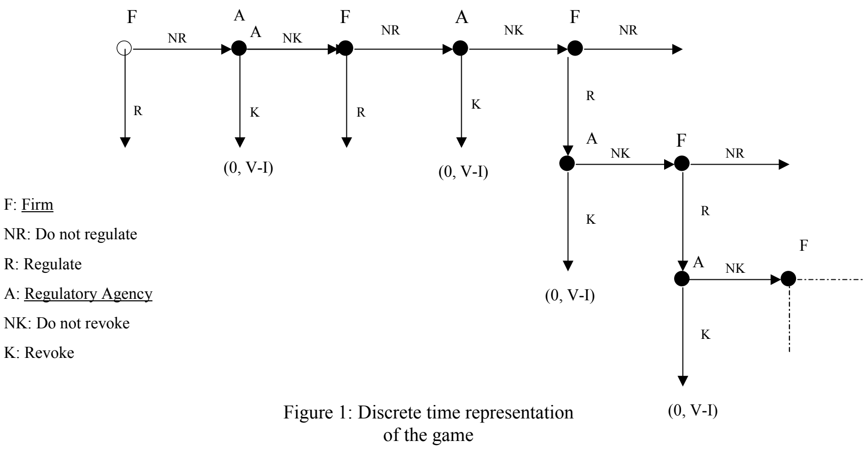
F
A
F
A
F
NR
A
NK
NR
NK
NR
R
K
R
K
R
(0, V-I)
(0, V-I)
A
NK
F
NR
F: Firm
NR: Do not regulate
R: Regulate
A: Regulatory Agency
NK: Do not revoke
K: Revoke
K
R
(0, V-I)
F
A
NK
K
(0, V-I)
Figure 1: Discrete time representation
of the game

Figure 1:

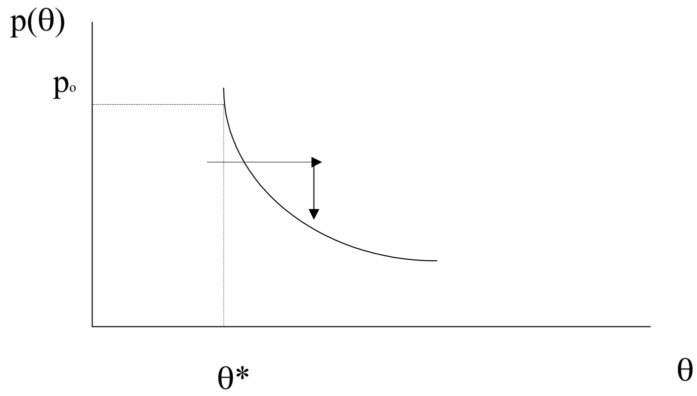

Figure 2: Price regulation

Figure 2:

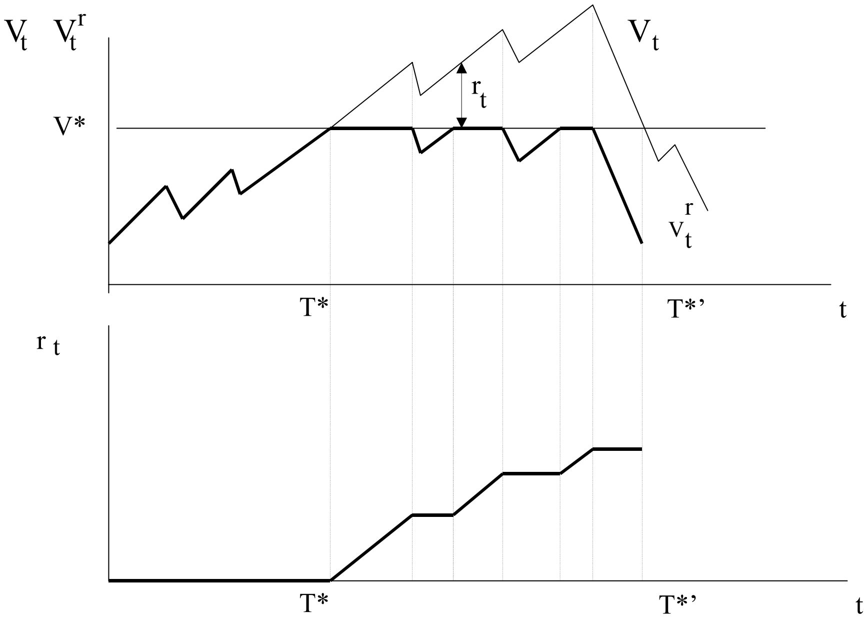

Figure 3: The regulatory timing

Figure 3: